Large Hierarchies from Attractor Vacua

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Abstract

We discuss a mechanism through which the multi-vacua theories, such as String Theory, could solve the Hierarchy Problem, without any UV-regulating physics at low energies. Because of symmetry the number density of vacua with a certain hierarchically-small Higgs mass diverges, and is an attractor on the vacuum landscape. The hierarchy problem is solved in two steps. It is first promoted into a problem of the super-selection rule among the infinite number of vacua (analogous to θ-vacua in QCD), that are finely scanned by the Higgs mass. This rule is lifted by heavy branes, which effectively convert the Higgs mass into a dynamical variable. The key point is that a discrete "brane-charge-conjugation" symmetry guarantees that the fineness of the vacuum-scanning is set by the Higgs mass itself. On a resulting landscape in all, but a measure-zero set of vacua the Higgs mass has a common hierarchically-small value. In minimal models this value is controlled by the QCD scale and is of the right magnitude. Although in each particular vacuum there is no visible UV-regulating low energy physics, the realistic models are predictive. For example, we show that in the minimal case the "charge conjugation" symmetry is automatically a family symmetry, and imposes severe restrictions on quark Yukawa matrices.

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1 Introduction

It is assumed usually that the solution to the Hierarchy Problem requires the existence of some new physics around the TeV-scale, in order to regulate quadratic divergences in the Higgs mass. In the present article, we shall attempt to provide a counterexample to this statement. Our approach will be based on generalization of [1], which we shall closely follow. Another inspiration is provided by the recent progress [2, 3, 4, 5] in understanding the distribution of vacua on String Theory moduli spaces. We will comment on the connection with the latter work below.

In [1] it was shown that in simple "string inspired" extensions of the standard model, postulating the coupling to the heavy branes and the three-form fields, the number of vacua with hierarchically small value of the Higgs VEV *diverges* due to symmetry reasons. Such a value of the Higgs VEV was called an "attractor" point in the space of vacua. The crucial point is that by symmetry the attractor point is stable against the quantum corrections. The situation is rather peculiar. The theory in question has an infinite number of vacuum states separated by the large potential barriers. The Higgs mass (and the VEV) takes different values in different vacua, but the vacua with the large values of the Higgs VEV are rare and gradually increase in number density towards the smaller Higgs VEV. An infinite number density of vacua cluster around a certain hierarchically small value of the Higgs VEV, which marks the attractor point. In the other words, the scanning of the vacuum landscape by the Higgs mass becomes *hyper-fine* near the attractor value.

In each particular vacuum within the neighborhood of the attractor point, there is no UV-regulating new physics around the scale of the Higgs mass. So a naive observer, living inside of any such vacua and suspecting nothing about the multiplicity of the similar vacuum states, would attribute the smallness of the weak interaction scale to a mysterious fine tuning. Nevertheless, for an "outside" observer, knowing that the number density of such vacua is divergent, the "fine-tunning" becomes perfectly natural. The Hierarchy Problem is solved since the prior probability distribution is singular, and probability is sharply peaked around the vacua with the hierarchically small Higgs mass.

These ideas indicate that multi-vacua theories, such as String Theory, have a potential of solving the Hierarchy Problem without invoking any UV-stabilizing new physics around the weak scale. Hence, in such theories our ideas about naturalness must be reconsidered. This view is supported by the recent progress in the field.

It is becoming evident [6, 2, 7, 8, 9, 3, 4, 5, 10] that the String Theory "landscape"[8], which reveals enormous complexity of the vacuum states, could play an important role in selection of the observed vacuum. So it is conceivable that the attractor solution of the hierarchy problem could find a natural implementation in String Theory. Putting aside the question of the Higgs mass and the strength of the divergence in the vacuum number density, the attractor of [1] can be viewed as some sort of a "holographic dual" to one class of vacua considered by Giddings, Kachru and and

Polchinski [2], which arise at conifold points in Calabi-Yau moduli space[2]. The recent interesting studies[3, 4, 5] indicate that "attractive" conifolds with flux can lead to clustering of high number of vacua. Although the clustering of the vacua in case of conifolds is weaker than in case of [1], it will be shown below that this feature is rather parameter-dependent. This connection deserves further study [11]. Some related ideas can also be found in [10].

Although, in the present paper we shall limit ourselves with effective field theory analysis of the attractor idea, string theory concepts will play the crucial role in this analysis. We discuss how the attractor behavior arises in effective field theory that involves some of the key ingredients of string compactifications, such as the three-form fields, axions (two-forms) and effective 2-branes. We shall show that, although the attractor solution does not require any UV-regulating new physics, nevertheless the realistic models have a predictive power and are potentially testable. For example, in the most minimal case, without any low energy extension of the Standard Model, we show that the attractor-stabilizing symmetry is automatically a *family* symmetry of Standard Model quarks, and implies restrictions on the structure of quark Yukawa couplings.

Among the possible approaches to the Hierarchy Problem, the attractor solution is probably the closest possible analog of the axion solution of the Strong CP problem. Indeed, from the beginning of our treatment the Hierarchy Problem, from being a problem of the UV-*stability* of the Higgs mass (m_ϕ), gets promoted into the problem of a *super-selection rule* among the infinite vacua scanned by m_ϕ. These are analogous to θ-vacua in QCD that are scanned by the θ parameter[13]. The advantage of dealing with a super-selection problem, as opposed to the one of UV-stability is the following. Unlike the latter problem, the new physics that solves the former can be arbitrarily weakly coupled to the Standard Model sector. The famous example of such new physics in case of the Peccei-Quinn (PQ) solution[12] of the Strong CP problem is the axion [14, 15], that can be arbitrarily weakly coupled and practically invisible [16, 17]. As we shall see, in our case too, the new physics that solves the Hierarchy Problem via attractor mechanism can be practically decoupled from the Standard Model sector. Of course, along with the analogies, there are fundamental differences with the axion mechanism. In both cases the parameter of interest is promoted into a dynamical variable. In PQ case this is the θ-angle, and in our case the Higgs mass m_ϕ. In both cases, the vacua with small values of the parameters are selected, but the selection mechanisms are different. In case of the θ-angle the selection is via vacuum relaxation mechanism, since small θ corresponds to the true groundstate of the system[20]. In our case, the selection happens through the enormous multiplicity of states. Because of attractor, in all, but a measure-zero number of vacua, the Higgs mass is small.

The paper is organized as follows. In section 2, we briefly summarize our criteria

[2]We thank Shamit Kachru for pointing out the possible connection, and for enlightening discussions. We also thank Michael Douglas for comments about the possible connection with his work [4].

of naturalness and the essence of the attractor vacuum. In section 3, we gradually build the attractor vacuum, by putting all essential ingredients together. At the end of this section we derive an effective equation for finding the vacua on the landscape, and give their number counting. In section 4, we give a general discussion of the shift of the attractor point to the realistic values of the Higgs VEV. The precise mechanisms are discussed in sections 5 and 6. In section 5, we study the effect of world volume terms, such as brane-localized mass terms, on the attractor dynamics, and show that depending on the parameters they can either lead to sharpening of the attractor, or to regulating it (that is, cutting-off the divergence in vacuum number densities). In section 6, we discuss the realistic model building and present the two versions of the complete models. We show that the minimal one, that requires no enlargement of the electro-weak sector imposes severe restrictions, on the structure of Yukawa matrices. The next to minimal case, which is less constraining is discussed at the end of section 6. In section 7, we show that when extended to grand unified theories, the attractor solution automatically solves the problem of doublet-triplet splitting. In appendix A, we show how the theories with attractor can be obtained from local gauge-invariant theories after integrating out the Stückelberg field, and in appendix B we discuss some related potential issues. In appendix C, we discuss some exact solutions. In appendix D, we show, following [1], that the branes charged under the three-form fields can be the axionic domain walls. Finally in appendix E, we study how the axionic walls with field-dependent charges could be obtained without the Stückelberg method, by integrating out some intermediate scalars.

2 Naturalness and the Essence of the Attractor Phenomenon

In this section, we wish to briefly formulate our criteria of naturalness, and summarize the essence of the attractor solution. The Hierarchy Problem is the problem of UV-sensitivity of the Higgs mass (m_ϕ) and consequently of the Higgs VEV ($\langle\phi\rangle$). The excellent formulation of the problem can be found in [18]. The attractor mechanism solves this problem by selecting the vacuum with the hierarchically small Higgs mass. The selected value is UV-*insensitive* due to the symmetry reasons. In this respect, the attractor solution of the Hierarchy Problem is natural in the same sense as the Technicolor[18, 19], or the low energy supersymmetry, but has some advantages over the latter. For example, as we shall show, the attractor mechanism also automatically solves the problem of Doublet-Triplet Splitting in grand unified theories, which supersymmetry alone fails to solve.

By simple choice of symmetry, the attractor solution gives possibility to obtain the Higgs VEV in terms of the QCD scale.

The essence of the attractor solution can be summarized as follows. Attractor is a special point on the energy landscape that contains an infinite number of discrete vacua. The vacua are scanned by a given parameter, which in the case of interest is

the Higgss mass. What makes the attractor landscape special is the fact that, due to the symmetry reason, all, but the measure-zero set of the vacuum states exhibit practically-equal and hierarchically-small values of the Higgs mass. This value is the attractor point.

The attractor vacuum is achieved in two steps. First, by coupling the Higgs to the field strength of an antisymmetric three-form, we promote the Hierarchy Problem into the problem of the super-selection rule, analogous to the Strong CP problem in QCD. The vacua are scanned by the Higgs mass m_ϕ, which plays the role analogous to θ-angle that scans the QCD-vacua. The super-selection rule is lifted by 2-branes that effectively *source* the Higgs mass, and allow the quantum transition between the vacua with different m_ϕ. Thus, the Higgs mass gets promoted into a dynamical variable. This shares some analogy with the axion solution of the Strong CP Problem, in which θ-angle gets promoted into a dynamical variable. However, the reason why the desired vacuum is selected in the attractor case is different from the axion scenario. In the latter case, because the dynamical θ-angle changes continuously, the selection is energetic. The selected vacuum is the true ground-state of the theory, to which the system relaxes on a microscopic time scale. In our case, the transitions between the vacua with different m_ϕ are discretized. The barrier between the vacua is very large, but the fineness of m_ϕ- scanning changes throughout the ladscape, and becomes super-fine around the attractor point. Because, of the large potential barrier, each vacuum is extremely long lived. In such a situation energetics plays no role in selecting the vacuum. Instead what is important is the density of the vacua with the given values of m_ϕ. We show that due to symmetry reason, which triggers a profound back reaction on the brane charge, essentially all vacua, cluster around a certain hierarchically-small value of m_ϕ, which is the attractor point.

3 The Attractor Vacuum

3.1 The Super-Selection Rule

Expanding the analysis of [1], we start with a detailed discussion of the "attractor" idea. String Theory contains various antisymmetric form fields, which after compactification to four dimensions give rise to three-forms, two-forms (axions) and one-forms (vectors). The crucial role in our solution of the hierarchy problem is played by the three-form field $C_{\alpha\beta\gamma}$. For a free three-form field the lowest order parity-invariant action has the following form

$$\int_{3+1} \frac{1}{48} F_{\mu\alpha\beta\gamma} F^{\mu\alpha\beta\gamma} \tag{1}$$

where $F_{\mu\alpha\beta\gamma} = d_{[\mu} C_{\alpha\beta\gamma]}$ is the four-form field strength. This action is invariant under the gauge transformation

$$C_{\alpha\beta\gamma} \to C_{\alpha\beta\gamma} + d_{[\alpha} \Omega_{\beta\gamma]}, \tag{2}$$

where Ω is a two-form. Because of this gauge freedom in four dimensions C contains no propagating degrees of freedom, and its equation of motion

$$\partial^{\mu} F_{\mu\nu\alpha\beta} = 0 \qquad (3)$$

is solved by

$$F_{\mu\nu\alpha\beta} = F_0 \, \epsilon_{\mu\nu\alpha\beta}, \qquad (4)$$

where F_0 is an arbitrary constant. Hence, in the absence of other interactions, the effect of the 3-form is reduced to adding an arbitrary integration constant to the Lagrangian. This constant will contribute to the overall cosmological term. However, in the presence of interactions with the other fields, the integration constant F_0 will also contribute to their effective masses and couplings. Consider, for instance, an interaction with a scalar field ϕ. In what follows we shall treat ϕ as the prototype of the Standard Model Higgs. The lowest order parity and gauge-invariant Lagrangian describing a non-trivial interaction between ϕ and C has the following form

$$L = |\partial_{\mu}\phi|^2 - \frac{1}{48}F^2 + |\phi|^2 \left(m^2 + \frac{F^2}{48M^2} \right) - \frac{\lambda}{2} |\phi|^4 + ... \qquad (5)$$

Here λ is the quartic coupling, and m and M are the mass parameters, that naturally are of the order of the UV cut-off, which we shall take to be around the Planck mass M_P. For definiteness, we shall assume $m^2 > 0$, $M^2 > 0$. Note that the value of F determines the value of the effective mass^2 and consequently the VEV of the Higgs field. The latter is equal to

$$|\phi|^2 = \frac{1}{\lambda}(m^2 + F^2/48M^2) \qquad (6)$$

The value of F is determined from the equation of motion

$$\partial^{\mu} \left((1 - |\phi|^2/M^2) \, F_{\mu\nu\alpha\beta} \right) = 0, \qquad (7)$$

which is solved by

$$F_{\mu\nu\alpha\beta} = \frac{F_0 \, \epsilon_{\mu\nu\alpha\beta}}{(1 - |\phi|^2/M^2)} \qquad (8)$$

where F_0 is an arbitrary integration constant. Plugging this solution into the equation for the Higgs field we get the following effective equation determining the Higgs VEV

$$\left(-m^2 + \frac{F_0^2}{2M^2(1 - |\phi|^2/M^2)^2} \right) \phi + \lambda \, |\phi|^2 \phi = 0 \qquad (9)$$

It is obvious that the above theory has a continuum of the vacuum states, labeled by F_0. In many of these vacua the VEV and the mass of the Higgs is much smaller than the cut-off. These are the vacua with $m^2 - F_0^2/2M^2 \ll M^2$, in which $\phi \ll M$. For instance, there is a vacuum with $F_0^2 = 2m^2M^2$ in which $\phi = 0$. In this vacuum ϕ is exactly massless.

Although, there exist vacua with a light scalar, the hierarchy problem is nevertheless *not* solved in the above theory. The reason is twofold. First in the above theory the vacua are uniformly scanned by the integration constant F_0, and the light Higgs vacua are not special in any way. Secondly, F_0 is not a dynamical quantity, and there is no transition between the different vacua. In the other words there is a super-selection rule in F_0, no vacuum is preferred over any other, and any choice of F_0 is good. In this respect F_0-vacua are similar to theta-vacua in QCD[13]. As it is well known, in QCD with no massless quarks there is a continuum of *physically distinct* vacuum states, that can be parameterized by a periodic variable θ. These θ-vacua obey the super-selection rule, there is no transition between the states with different θ. This situation gives rise to celebrated Strong CP Problem, since the phenomenologically acceptable vacua, with $\theta < 10^{-9}$ are not particularly preferred by the system.

The situation in our case is analogous. The model (5) has a continuum of the vacuum states, scanned by F_0 or equivalently by $\langle\phi\rangle$, and there is no transition between the different $\langle\phi\rangle$-vacua. Thus, the hierarchy problem, from the problem of UV-instability of the Higgs mass, got promoted into the super-selection problem, analogous to the Strong CP Problem in QCD.

In order to solve the former, we shall try to follow, as closely as possible, the general strategy adopted by the axion solution of the Strong CP Problem. As it is well known, this solution is based on, *a)* first promoting θ into a dynamical variable (axion), and *b)* showing that $\theta = 0$ is the true groundstate of the system.

Thus, in order to solve the hierarchy problem we need to accomplish the two more steps.

1) Promote F_0 into a dynamical variable;

2) Find the symmetry reason that will ensure that the vacua are not uniformly distributed and the vacua with the light Higgs are preferred over all the others.

As we shall see, the first step is achieved by introduction of branes which permit the transition between the different ϕ-vacua. The difference with the axion case is that the transition is quantum-mechanical and discrete, as opposed to being classical and continuous. This circumstance creates a profound difference between our solution of the Hierarchy Problem and the axion solution of the Strong CP one. Impossibility of the continuous classical transition tells us that we cannot directly generalize the axion mechanism of θ-relaxation, but it also suggests a natural substitution. Indeed, because there is no classical transition, the energy difference is unimportant for selecting the vacuum state, and what matters is the multiplicity of vacua with a given value of ϕ. We shall see the hierarchy problem is solved, because infinite number of vacua cluster around the small ϕ-value, by the symmetry reason.

In what follows we shall give a detailed discussion of this phenomenon.

3.2 The Role of the Branes: Breaking the Super-Selection

As said above, the super-selection rule is lifted by introducing 2-branes (membranes) that source our three-form field $C_{\alpha\beta\gamma}$. Ignoring the Higgs field for a moment, the effective action incorporating the interaction with branes can be written as

$$\frac{q}{6}\int_{2+1} d^3\xi\, C_{\mu\nu\alpha}\left(\frac{\partial Y^\mu}{\partial\xi^a}\frac{\partial Y^\nu}{\partial\xi^b}\frac{\partial Y^\alpha}{\partial\xi^c}\right)\epsilon^{abc} - \int_{3+1}\frac{1}{48}F^2, \tag{10}$$

where the first explicitly-written term describes the interaction between the brane and the three-form. q is the charge of the brane, and $x^\mu = Y^\mu(\xi)$ specify a $2+1$-dimensional history of the brane in 3+1-dimensions as a function of its world-volume coordinates ξ^a $(a = 0, 1, 2)$. This term can be rewritten in form of the following four-dimensional integral

$$\int d^4x\,\frac{1}{6}J^{\alpha\beta\gamma}C_{\alpha\beta\gamma} \tag{11}$$

where $J^{\alpha\beta\delta}$ is the brane current

$$J^{\alpha\beta\gamma}(x) = \int d^3\xi\delta^4(x - Y(\xi))\,q\left(\frac{\partial Y^\alpha}{\partial\xi^a}\frac{\partial Y^\beta}{\partial\xi^b}\frac{\partial Y^\gamma}{\partial\xi^c}\right)\epsilon^{abc} \tag{12}$$

Obviously, the current $J_{\alpha\beta\gamma}$ is conserved as long as q is a constant.

The brane self-action has the standard form

$$-T\int d^3\xi\sqrt{-g}, \tag{13}$$

where T is the brane tension (a mass per unit surface), and $g_{ab} = \partial_a Y^\mu \partial_b Y^\nu \eta_{\mu\nu}$ is the induced metric on the brane. Note that, since the bulk 4-dimensional gravity plays no essential role in our considerations, we have taken a flat Minkowskian 4-dimensional metric $\eta_{\mu\nu}$. Despite this, the induced metric on the brane is not flat in general, due to the dynamical curving of the brane. With the brane source taken into the account, the equation of motion of the 3-form now becomes,

$$\partial_\mu F^{\mu\nu\alpha\beta} = -q\int d^3\xi\delta^4(x - Y(\xi))\left(\frac{\partial Y^\nu}{\partial\xi^a}\frac{\partial Y^\alpha}{\partial\xi^b}\frac{\partial Y^\beta}{\partial\xi^c}\right)\epsilon^{abc} \tag{14}$$

The brane can be taken to be flat and static, $Y^\mu = \xi^\mu$ for $\mu = 0, 1, 2$, and $Y^3 = 0$. The equation of motion then simplifies to

$$\partial_\mu F^{\mu\nu\alpha\beta} = -q\delta(z)\epsilon^{\nu\alpha\beta z} \tag{15}$$

where $z = 0$ is the location of the brane. Both (14) and (15) show that the brane separates the two vacua in either of which F_0 is constant, and the two values differ by $|q|$. Thus, the introduction of branes ensures that the transition between the vacua with different values of F_0 is possible, as long as the value of F_0 changes by the integer multiple of q. Hence the discrete quantum transition between the

different vacua are possible via nucleation of closed branes (this fact was used in an interesting attempt[21] to explain the smallness of the cosmological term).

In the other words, the theory given by the action (10) has multiplicity of vacua that can be labeled by an integer n. The value of the field strength in this vacua is

$$-\frac{1}{24}F_{\alpha\beta\gamma\mu}\epsilon^{\alpha\beta\gamma\mu} \,=\, F_0 \,=\, qn \,+\, f_0, \tag{16}$$

where f_0 is a constant, which we will set equal to zero. That is, the value of F is quantized in units of the brane charge . Restoring the coupling to the Higgs field, the equation (9) determining the Higgs VEV now becomes

$$\left(-m^2 \,+\, \frac{(nq)^2}{2M^2(1\,-\,\frac{|\phi|^2}{M^2})^2}\right)\phi \,+\, \lambda\,|\phi|^2\phi \,=\, 0 \tag{17}$$

The good news is that now the transition between the vacua with different n are possible, however the hierarchy problem is still not solved. First, because both m and M are large, we need a very small q in order to ensure a fine enough scanning of the Higgs mass. Secondly, even for a small q, the vacua with a small Higgs VEV are not preferred over the others. Both problems can be cured in one shot, by requiring a symmetry which will promote q into a continuous function of ϕ. For example,

$$q \,\rightarrow\, q(\phi) \,\propto\, \phi^N \tag{18}$$

Then, the zero of the function $q(\phi)$ will become an "attractor" (accumulation of infinite number of vacua) in the space of vacua. This will guarantee both the super-fine scanning of the Higgs mass and the preference of the vacua with the small values of the Higgs VEV. Thus, idea of the "attractor" is that the multiplicity of the vacua with the small values of the Higgs VEV become divergent (or at least very sharply peaked), because of the symmetry reasons.

The essence of the attractor phenomenon can be summarized schematically in the following sequence (written in units of the fundamental scale)

$$(q = \phi^N) \,\rightarrow\, (\Delta F = q) \,\rightarrow\, \left(\frac{\Delta\phi}{\phi} \sim q\right) \,\rightarrow\, (\Delta q \sim q\phi^{N-2}) \tag{19}$$

The arrows indicate that a non-zero charge of the brane separating the two different vacua implies the change of F across the brane, which implies the change of ϕ. The latter implies the change of the brane charge in a new vacuum, and closes the cycle. We shall give a detailed discussion of the dynamics of the above sequence throughout the paper.

3.3 Charge Conjugation: Creating an Attractor

Thus, to achieve an attractor, we must promote the brane charge q into the function of ϕ, such that q vanishes for small values of ϕ. To guarantee this, we shall require a new discrete symmetry Z_{2N}

$$\phi \rightarrow e^{i\frac{\pi}{N}}\phi, \tag{20}$$

which acts on the brane as "charge conjugation". That is, we require that $q \to -q$ under Z_{2N}. The invariance under Z_{2N} then demands q to be an odd function of ϕ^N. The simplest choice is

$$q \,=\, q_{eff}(\phi) \,=\, \frac{\mu}{2}\left(\frac{\phi}{M_P}\right)^N \,+\, h.c.. \tag{21}$$

where μ is some constant.

For successful implementation of the above idea, we need to address the following technical issue. For non-constant q the current (12) is not conserved, and hence the coupling (11) is not gauge invariant. In order to maintain the gauge invariance, following [1], we shall modify this coupling in the following way.

$$-\int d^4x\,\frac{1}{6}\,C_{\alpha\beta\gamma}J^{\alpha\beta\gamma}_{(T)} \tag{22}$$

where $J_{(T)}$ is the transverse part of the current

$$J^{\alpha\beta\gamma}_{(T)} \,=\, \frac{1}{6}\,\Pi^{[\alpha}_{\mu}J^{\mu\beta\gamma]}. \tag{23}$$

Here $\Pi_{\mu\nu} \,=\eta_{\mu\nu} \,-\, \frac{\partial_\mu\partial_\nu}{\partial^2}$ is the transverse projector. [3]

For constant q, we have $\partial^\alpha J_{\alpha\beta\gamma} \,=\, 0$ and $J_{(T)} \,=\, J$. Thus, the coupling (22) reduces to (11). This fact accomplishes our goal. In each given vacuum the expectation value of Higgs is fixed, and so q is constant. So in each vacuum with unexcited Higgs field the brane couples to the three-form in a normal way. On the other hand, the change of q from vacuum to vacuum is permitted, because $C_{\alpha\beta\gamma}$ only couples to the transverse part of $J_{\alpha\beta\gamma}$. The existence of the attractor point at $\phi \,=\, 0$ is guaranteed by the fact that $J_{(T)} \to 0$ when $\phi \to 0$.

The coupling (22) is the gauge-invariant generalization of (11) for the case of a non-constant charge $q(\phi)$. For the constant ϕ the above coupling is equivalent to (10) with $q_{eff} \,=\, \mu\,Re(\phi/M_P)^N$. Although, the coupling (22) contains a projector, it actually can be obtained from a local underlying theory after integrating out certain degrees of freedom. This issue is discussed in the Appendix A. It is shown there how the coupling (22) can be obtained by integrating out the Goldstone-type degrees of freedom ala Stückelberg. In a very crude sense, the Stückelberg field plays the role analogous to the one of axion in the solution of the Strong CP problem. Some alternatives to Stückelberg method will be discussed in [22] [4].

Putting all the ingredients together let us now show that with Z_{2N}-symmetry, theory has an attractor point in the space of vacua at $\phi = 0$. For the convenience

[3] $J_{(T)}$ can also be written in the following form $J^{\alpha\beta\gamma}_{(T)} \,=\, \frac{1}{6}\,\epsilon^{\alpha\beta\gamma\mu}\Theta^{\nu}_{\mu}\epsilon_{\nu\tau\rho\omega}C^{\tau\rho\omega}$, where $\Theta_{\mu\nu} \,=\, \frac{\partial_\mu\partial_\nu}{\partial^2}$ is the longitudinal projector.

[4] We thank Gregory Gabadadze for enlightening discussions on this and other issues.

we write down the combined Lagrangian, which takes the following form

$$
\begin{aligned}
L \;=\; & |\partial_\mu \phi|^2 \,-\, \frac{1}{48} F^2 \,+\, |\phi|^2 \left(m^2 \,+\, \frac{F^2}{48 M^2} \right) \,-\, \frac{\lambda}{2}\, |\phi|^4 \\
& - \frac{1}{6}\, C_{\alpha\beta\gamma} J^{\alpha\beta\gamma}_{(T)}
\end{aligned}
\tag{24}
$$

The above theory admits the divergent number of vacua at small VEV of ϕ, at least when[5]

$$
m \gg \frac{\mu}{\sqrt{2} M} \left(\frac{m}{\sqrt{\lambda} M_P}\right)^N. \tag{25}
$$

In order to see this, we shall integrate out the brane and the 3-form field and write down the effective potential for ϕ. Choosing the brane to be located at $z = 0$, the equations are

$$
\partial^\mu \left((1 - |\phi|^2/M^2) F_{\mu\alpha\beta\gamma} \right) \;=\; \mu \epsilon_{\alpha\beta\gamma\nu} \Theta^{\nu z} \left[Re \left(\frac{\phi}{M_P} \right)^N \, \delta(z) \right] \tag{26}
$$

$$
\partial^2 \phi \,-\, \left(m^2 \,+\, \frac{F^2}{48 M^2} \right) \phi \,+\, \lambda\, |\phi|^2 \phi \,+\, \frac{\mu N}{12} \frac{\phi^{*N-1}}{M_P^N}\, \delta(z)\, \Theta_{z\nu} \epsilon^{\nu\alpha\beta\gamma}\, C_{\alpha\beta\gamma} \;=\; 0 \tag{27}
$$

In integrating these equations, for simplicity, we will first make the following approximation which is well justified in the vacua with the small VEVs of ϕ (more rigorous derivation is discussed in the Appendix C).

1) We shall ignore terms of order ϕ^2/M^2 in the gauge-kinetic function of F in l.h.s. of equation (26);

2) We shall ignore the terms proportional to the derivatives of q_{eff} on r.h.s. of (26).

The above is justified, because the change in q in each elementary transition between the small ϕ vacua is very small. So the correction to ΔF_0 because of change in q is of the higher order smallness in ϕ/M_P, and can be ignored.

Indeed, change in ϕ ($\Delta\phi$), in each elementary step that connects the neighboring vacua, is due to change of F, which is $\Delta F \simeq q_{eff}$. Because the VEV of ϕ in any vacuum is given by (6), the change in ϕ is

$$
\Delta(\phi^2) \,\simeq\, \frac{\Delta F_0 F_0}{\lambda M^2} \,\simeq\, q_{eff} \frac{F_0}{\lambda M^2} \tag{28}
$$

[5]The meaning of this constraint is to avoid "overshooting" to the vacua with the large positive Higgs mass, in case if we start in a vacuum with $F = 0$, in which the Higgs VEV is maximal, and so is the brane charge.

Thus, even if the attractor point happens to be for the maximal value $F \sim M^2$, the change $\Delta\phi^2 \sim q_{eff}$, and consequently

$$\Delta q_{eff} \,=\, \frac{N\mu}{2}\frac{\phi^{N-2}}{M_P^N}\Delta(\phi^2) \,\sim\, q_{eff}\,\frac{\mu\phi^{N-2}}{M_P^N} \tag{29}$$

Thus, the change in the brane charge is higher order in ϕ/M_P. So around the attractor point ($\phi \to 0$) this subleading correction can be safely ignored in each elementary step and q_{eff} can be regarded as constant. Only after many steps the accumulated change in ϕ can become significant.

This fact simplifies the equation (26). Integrating it for the constant q_{eff} we get that F is given by (4) with

$$F_0 \,=\, nq_{eff}, \tag{30}$$

where n is an integer that labels the different vacua. Plugging this result into the equation for ϕ we get the effective equation defining the VEV of ϕ [6]

$$-\left(m^2 - n^2\frac{\mu^2}{2M^2}\left(\frac{\phi^N}{M_P^N}\right)^2\right)\phi \,+\, \lambda\,\phi^3 \,=\, 0 \tag{31}$$

Thus, n labels different vacua. It is obvious that there are infinite number of vacua close to $\phi = 0$. Thus, vacuum with a vanishing VEV is an *attractor*.

We should stress that Z_{2N}-symmetry guarantees the UV-stability of the attractor point. Indeed, the attractor point is the vacuum in which the brane charge vanishes $q = 0$. Due to Z_{2N}-symmetry, this happens when $\phi = 0$. Thus, any renormalization of ϕ implies the corresponding renormalization of the brane charge, such that $\phi = 0$ remains an attractor point.

3.4 Counting the Number of Vacua

We shall now give a simple general rule for counting the number of vacua near the attractor point. For simplicity we set the parameter $\lambda \sim 1$. We wish to estimate the number of vacua in which the expectation value of the Higgs field is of the order of a given value $\phi_0 \ll M$. The number of the vacua in which the Higgs VEV is $\phi \sim \phi_0$, we shall denote by n_{ϕ_0}. There is a simple way to estimate this number. We start with a vacuum with a given VEV $\phi = \phi_0$ and begin lowering ϕ by jumping to new vacua via creating branes, till the VEV changes in first non-zero digit. The VEV of ϕ in each vacuum is given by (6). The change of ϕ in each elementary step is given by (28) and is small, because q is small. The number of vacua n_{ϕ_0} is equal to the number of steps that will make the accumulated relative change of ϕ of order one. That is n_{ϕ_0} is defined by the condition that the quantity

$$\frac{\Delta\phi}{\phi_0} \sim \frac{qn_{\phi_0}}{\phi_0^2} \tag{32}$$

[6]Without the loss of generality we can put VEV of ϕ to be real

should become of order one. Thus, the number of vacua is

$$n_{\phi_0} \sim \frac{\phi_0^2}{q} \sim \frac{M_P^N}{\mu\phi_0^{N-2}}, \tag{33}$$

which for $N > 2$ diverges as $\phi_0 \to 0$.

The above expression should not create a false impression that there is no attractor for $N = 2$ or less. The equation (33) indicates that the number of vacua in which ϕ is of the order of a given value ϕ_0 diverges as $\phi_0 \to 0$, as long as $N > 2$. For smaller N, this number is either constant or decreases with ϕ_0, but the total number of vacua accumulated around $\phi = 0$ is always infinite!

In the other words, the number of vacua with the Higgs VEV in an interval $M_P > \phi > \phi_0$ is given by the sum

$$n_{(>\phi_0)} = \sum n_{\phi_0} \tag{34}$$

where the sum is taken over all the discrete vacua. As said above, for the condition (25) the vacua can be arranged into the groups labeled by ϕ_0. In any vacuum belonging to ϕ_0-th group the VEV of the Higgs is equal to ϕ_0 in the first nonzero digit, and the number in each group is given by n_{ϕ_0}. Although for $N = 2$ or less this number either stays constant or decreases with ϕ_0, the density of the groups grows as $1/\phi_0^2$ and over-compensates the decrease of n_{ϕ_0} for any $N > 0$. Putting it shortly, for the small ϕ_0 the sum can be approximated by the following integral

$$n_{(>\phi_0)} \sim M_P \int_{\phi_0}^{M_P} n_\phi \frac{d\phi}{\phi^2} \tag{35}$$

which shows, that even for $N = 1$ the total number of vacua diverges as $\log(\phi_0)$. The divergence of $n_{(>\phi_0)}$ for $\phi_0 \to 0$ for $N > 0$, can also be seen from (31), which for $N = 1$ gives the following expression for the VEV of ϕ

$$|\phi_0|^2 = \frac{m^2}{\lambda + n^2(\mu^2/2M^2M_P^2)}. \tag{36}$$

This VEV approaches $\phi_0 = 0$ for $n \to \infty$.

4 The Weak Scale Shift of the Attractor

We have shown that a theory in which the brane charge q is set by the VEV of a scalar field has a divergent number of vacua with the values of the scalar field for which $q \to 0$. This is the essence of the attractor phenomenon. In the above example the "charge conjugation" Z_{2N} symmetry guaranteed that the attractor point was at $\phi = 0$. However, in order to solve the hierarchy problem in the Standard Model we have to make sure that the attractor point is not at zero, but instead at the observed value $\phi \sim 100$GeV. As it was shown in [1], when the attractor theory is considered

in the cosmological context of an eternally inflating Universe, the attractor point can be shifted to the value of the Hubble parameter during inflation. The solution of the hierarchy problem then would require this parameter to be around the weak scale or so.

Despite the cosmological possibilities, it is important to have other mechanisms that could generate the small shift of the attractor point in a cosmology-independent way. One universal possibility is to shift the brane charge by a small ϕ-independent amount

$$q_{eff} \rightarrow \frac{\mu}{2}\left(\frac{\phi^N}{M_P^N} - \xi\right) + h.c.. \tag{37}$$

where ξ is a ϕ-independent part. We should think of q_0 as of "spurion", the VEV that spontaneously (softly) breaks Z_{2N} symmetry. It is important to stress that even for constant ξ, because $\xi = 0$ is an enhanced symmetry point, the value of ξ is *perturbatively-stable*. The attractor point then will be shifted to

$$\phi \,=\, (\xi)^{\frac{1}{N}} M_P \tag{38}$$

The origin of ξ is model dependent. We shall explore two possibilities. In section 5, we will show that ξ can come from the QCD condensate of the Standard Model quarks.[7] Another avenue, to be discussed in the next section, is to take into account the possible impact of brane-localized mass term of the Higgs field. As we will show, such terms can have non-trivial effects on the attractor dynamics.

5 The Effect of the Brane-Localized Potential

ϕ field may have various potential terms on the brane world volume, compatible with symmetries. The most important of these is a brane-localized mass term, which can be introduced in the four-dimensional action in the following form

$$-\int dx^4\, M_{br}(x)^2 |\phi|^2, \tag{39}$$

where

$$M_{br}^2(x) \,=\, \pm \int d\xi^3 \sqrt{-g}\, M_B \delta^4(x - Y). \tag{40}$$

In the above expression M_B is a positive mass parameter. The question is what is the effect of this brane-localized mass term on the attractor dynamics? We shall show that this effect depends on the sign in (66). For the positive sing the attractor at $\phi = 0$ becomes sharper, meaning that the number of vacua diverges faster for $\phi \rightarrow 0$. In the case of the negative sing, the two sub-regimes are possible, depending on the parameters. One possibility is that the attractor point is shifted away from zero, but the divergence of vacua is kept in tact. Another possibility is that the

[7] ξ can also be set by the expectation value of some hidden sector fermionic condensate, which has no direct interactions to Standard Model particles.

attractor becomes "softer", meaning that the brane charge cannot decrease below a certain minimal value. In the latter case, the scanning of the Higgs VEV cannot get finer beyond the certain minimal step, and correspondingly the divergence in the number density of vacua gets cut-off at some maximal value. We shall now give a more detailed discussion of the above two regimes.

5.1 A Positive Mass Term: Sharpening the Attractor

Consider the effect of the positive mass term first. Such a mass term, "repels" the ϕ field from the brane, and effectively diminishes its VEV at the brane location. Ignoring the effect of $\sim q_{eff}$-terms, the equation for ϕ in the background of the brane located at $z = 0$ is

$$\partial^2\phi \,-\, (\,m_{bulk}^2 \,-\, \delta(z)M_B)\phi \,+\, \lambda\phi^3 \,=\, 0 \tag{41}$$

where m_{bulk}^2 is the effective bulk mass term which includes the contribution coming from F

$$m_{bulk}^2 \,=\, m^2 \,+\, F_0^2/2M^2 \tag{42}$$

From (41) it is clear that the positive brane-localized mass term is seen by the field as a potential barries, and for $M_B \gg m_{bulk}$ the expectation value at the brane location $\phi(0)$ is considerably smaller than its bulk counterpart $\phi(\infty)$.

$\phi(0)$ can be estimated by minimizing the following expression (we ignore the factors of order one)

$$E \,=\, M_B\phi(0)^2 \,+\, (\phi(0) - m_{bulk})^2 m_{bulk} \,+\, (\phi(0)^2 \,-\, m_{bulk}^2)^2 m_{bulk}^{-1} \tag{43}$$

The first term in this expression comes from the brane mass term. The second and the third terms are the expenses in the gradient and the bulk potential energies. The full expression is minimized at

$$\phi(0) \sim \frac{m_{bulk}^2}{M_B} \tag{44}$$

Thus, in any given vacuum, the brane expectation value of ϕ is by the factor $\frac{m_{bulk}}{M_B}$ smaller then its bulk counterpart $\phi(\infty) \sim m_{bulk}$. Thus, for a given value of the bulk mass term, the value of the brane charge q_{eff} is by a factor of $(m_{bulk}/M_B)^N$ smaller, as compared to what it would be for $M_B = 0$. Correspondingly according to the general rule of vacuum counting, the number of vacua with a given VEV of the Higgs field $\phi \sim \phi_0$, becomes

$$n_{\phi_0} \sim \left(\frac{M_P^N}{\mu\phi_0^{N-2}}\right)\left(\frac{M_B}{\phi_0}\right)^N, \tag{45}$$

and is by factor of $\left(\frac{M_B}{\phi_0}\right)^N$ bigger than what it would be for $M_B = 0$. Thus, the positive sign brane-localized mass term makes the attractor stronger.

5.2 A Negative Mass: Smoothing Out the Attractor

Now let us show that in case of the negative sign the brane-localized mass term has an opposite effect, and may either shift the attractor or cutt off the divergence of the number of vacua. This is because for the negative brane mass term, ϕ develops a non-zero expectation value on the brane even for $m_{bulk}^2 = 0$ (that is, when the bulk VEV is zero). Thus, the brane continues to have a non-zero charge q even in $\phi = 0$ vacua, and the step of change stays finite. As a result number of vacua gets cut-off.

Let us show that for the negative sign of the brane mass term ϕ indeed develops a non-zero value on the brane in the limit $m_{bulk}^2 \to 0$. The fact that ϕ wants to condense on the brane can be seen by examining the linearized equation for small perturbations about the $\phi = 0$ solution in the brane background. This equation has the following form

$$\left(\partial^2 - \delta(z)M_B\right)\phi = 0 \tag{46}$$

It is obvious that there is a normalizable exponentially-growing tachyonic mode, localized on the brane

$$\phi = e^{\frac{1}{2}M_B t}e^{-\frac{1}{2}|z|M_B} \tag{47}$$

This instability signals that ϕ condenses on the brane and develops a non-zero expectation value there. This condensate is $\phi(0) \sim M_B$, since for $m_{bulk} = 0$, M_B is the only mass scale in the problem. Hence, the expectation value on the brane is $\sim M_B$ even though the bulk VEV vanishes.

This fact has profound implications for the attractor dynamics, since now even in the vacua with $m_{bulk} = 0$ the brane charge is non-zero and is given by

$$q_{min} \sim \mu(M_B/M_P)^N \tag{48}$$

To see what these implications are, first note that the value of F_0 in the vacuum with $m_{bulk} = 0$ is $F_0^2 = 2M^2m^2$. Then, if

$$M_B^2 \gg 4\sqrt{2}\frac{m}{M}q_{min}, \tag{49}$$

the attractor will be shifted to a positive value $m_{bulk}^2 = M_B^2/4$. In the opposite limit

$$M_B^2 < 4\sqrt{2}\frac{m}{M}q_{min} \tag{50}$$

the attractor will be regulated, and the divergence in the vacuum number density will be cut-off at

$$n_{max} \sim \frac{mM}{q_{min}} \tag{51}$$

The above two regimes can be understood from the fact that the equation

$$(\partial^2 - m_{bulk}^2 - \delta(z)M_B)\phi = 0 \tag{52}$$

has an exponentially growing normalizable tachyonic mode

$$\phi = e^{\frac{1}{2}t\sqrt{M_B^2+4m_{bulk}^2}} e^{-\frac{1}{2}|z|M_B}, \tag{53}$$

as long as [8]

$$M_B^2 + 4m_{bulk}^2 > 0. \tag{54}$$

Thus, only in this regime ϕ develops a non-zero VEV in the vicinity of the brane. If (50) holds in $m_{bulk} = 0$ vacuum, then the condition (54) will be violated within a single step, and the branes will become chargeless. Thus, q_{min} is the smallest possible non-zero brane charge. Due to this the singularity in the number density of vacua will get smooth-out at this point, according to (51).

6 Realistic model building and predictions

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6.1 The Need of a Second $SU(2) \times U(1)$-Doublet.

So far we have been discussing the attractor solution of the hierarchy problem on a toy example in which the prototype for the Standard Model Higgs was a complex singlet ϕ. In order to implement this idea in the realistic model, we have to promote ϕ into the doublet representation of $SU(2)\times U(1)$ group. This creates an issue of how to write down the gauge invariant interaction with the brane. Since $C_{\alpha\beta\gamma}$ carries no electroweak quantum numbers, $q(\phi)$ must be an $SU(2) \times U(1)$-invariant, but Z_{2N}-*odd* function of ϕ. This is, however, impossible to achieve by employing a single Higgs doublet, since the only possible gauge invariant $\phi^*\phi$ is also automatically an Z_{2N}-invariant. This in unacceptable, since Z_{2N}-*oddness* of the brane charge is what guarantees the UV-stability of the attractor point. Fortunately, there are number of ways for circumventing the above technicality. Here we shall discuss one of the simplest and economic ones.

6.2 Quark condensate as a second doublet

The QCD condensate of the light quarks in the Standard Model carries the quantum numbers identical to the Higgs doublet and contributes to the Higgsing of $SU(2) \otimes U(1)$-symmetry. In fact, because of this quark condensate, the electroweak W and Z bosons would get masses even in the absence of the Higgs scalar[18]. We can use this condensate in convolution with the Higgs doublet for creating the gauge-invariant (but Z_{2N}-*odd*) brane charge. There are two lowest possible gauge invariants (per

[8]For $M_B^2 + 4m_{bulk}^2 = 0$ there is a localized zero mode $\phi = \beta e^{-\frac{|z|M_B}{2}}$ that can be given an arbitrary expectation value without costing any energy up to bilinear in ϕ. In the other words, for $\lambda = 0$ there is a one-parameter class of the zero energy solutions with arbitrary β.

generation) that can be made out of the Higgs and the quark bilinears. These are ($SU(2)$-indexes are suppressed)

$$\phi \bar{Q}_L U_R \tag{55}$$

and

$$\phi^* \epsilon \bar{Q}_L D_R \tag{56}$$

respectively. Here Q_L is the left-handed quark doublet, and U_R, D_R are the right-handed up and dawn quarks respectively. ϵ is an antisymmetric $SU(2)$ tensor. Due to QCD quark condensate, the expectation value of these invariants is non-zero as long as $\phi \neq 0$. Thus, any of these invariants can be used for creating the attractor at $\phi = 0$. For this we have to require that the given invariant, e.g., (55) (or any power of it) is odd under the brane conjugation symmetry. For example,

$$(\phi \bar{Q}_L U_R) \rightarrow e^{i\frac{\pi}{N}}(\phi \bar{Q}_L U_R) \tag{57}$$

Then the brane charge becomes the function of (55)

$$q = \frac{\mu}{6} Re \left(\frac{\phi \bar{Q}_L U_R}{M_P^4} \right)^N \tag{58}$$

Note that since (55) transforms non-trivially under Z_{2N}-conjugation, it cannot appear in the Lagrangian. Thus, the Standard Model Yukawa coupling that could give a diagonal mass to the given U-quark is forbidden in this theory. This fact highlights a generic feature of the model, irrespective which quark condensate creates a brane charge, appearance of certain zeros in Yukawa matrix is the generic prediction. Precise structure depends on the Z_{2N} charge assignment, and will not be discussed here, but it is an important aspect for understanding the predictivity of the attractor solution.

6.3 A complete model

Putting all the ingredients together we can now write down a simple extension of the Standard Model which solves the hierarchy problem via the attractor mechanism. The action is

$$\begin{aligned} S \; = \; & \int_{3+1} |D_\mu \phi|^2 - \frac{1}{48} F^2 + |\phi|^2 \left(m^2 + \frac{F^2}{48 M^2} \right) - \frac{\lambda}{2} |\phi|^4 + \\ & - \frac{1}{6} C_{\alpha\beta\gamma} J^{\alpha\beta\gamma}_{(T)} - M_{br} |\phi|^2, \end{aligned} \tag{59}$$

plus the usual action of the Standard Model. In (59) D_μ is the covariant derivative. M_{br} is the brane-localized mass term given by (66), and we choose the sign to be positive. The current $J^{\alpha\beta\gamma}_{(T)}$ is given by (12) with

$$q_{eff} = \frac{\mu}{12} \left[\left(\frac{\phi \bar{Q}_L U_R}{M_P^4} \right)^N - \left(\frac{\bar{Q}_L U_R \bar{Q}_L D_R}{M_P^6} \right)^K \right] + h.c. \tag{60}$$

In this model we choose to use the quark condensate as a second Higgs doublet in order to construct a gauge-invariant brane charge. The attractor value is shifted away from $\phi = 0$ by the second quark condensate in the figure brackets. For the latter invariant to be non-zero the quarks must be taken from different generations. The integers N and K are determined by the transformation properties of the various fields under the discrete symmetry. This transformation properties also restrict the structure of Yukawa matrix elements. For instance, as said above for arbitrary N, the diagonal Yukawa coupling of the up quark is forbidden.

Taking all these terms into the account the attractor value for the bulk Higgs VEV is

$$\phi_{attr} \sim (M_B^{\frac{1}{2}} M_P^{\frac{2}{3}}) \left(\frac{\Lambda_{QCD}}{M_P}\right)^{(4\frac{K}{N}-2)} \tag{61}$$

Where Λ_{QCD} ~GeV is the strong interaction scale. For $M_B \sim M_P \sim 10^{19}$GeV, The correct attractor value is established around $K/N \simeq 5/7$ or so. Since the brane charge-conjugation symmetry implies nontrivial restrictions on the matrix of Yukawa couplings, it would be interesting to classify predictions of fermion mass relations for various assignments that lead to the correct attractor value.

We wish to notice that other Standard Model parameters, such as, for example, Yukawa coupling constants, can (and in general will) depend on the values of F and ϕ through some high-dimensional M_P-suppressed operators. The question then is how the attractor influences the values of such parameters. To answer this question it is useful to classify parameters by their transformation properties under Z_{2N}. The general rule then is that the parameters that are Z_{2N}-even do not change significantly near the attractor point, since in the zeroth order such parameters do not depend on the ϕ-VEV. The example of Z_{2N}-even parameters are, for instance, all the Yukawa coupling constants of the couplings that are allowed by the Z_{2N} symmetry. The dependence of such a coupling constant on ϕ and F can be parameterized in form of the expansion in series of invariants F^2/M_P^4 and $|\phi|^2/M_P^2$

$$g = g_0 + g_1 F^2/M_P^4 + g_2|\phi|^2/M_P^2 + ... \tag{62}$$

where g_0, g_1, g_2 are field-independent constants. It is obvious that unless g_0 is minuscule, the change of g near the attractor point is negligible, since both ΔF and $\Delta\phi$ vanish there. So generically, at the attractor point the expectation values of Z_{2N}-even parameters will be set by some attractor-insensitive physics.

6.4 The Heavy Higgs Doublet

An alternative to the quark condensate for creating and $SU(2) \times U(1)$-invariant brane charge, is to introduce a second Higgs doublet H. We shall assume that H has a positive mass square M_H^2 of order M_P^2, and has no expectation value in the bulk vacuum. However, H is allowed by symmetries to have a large brane-localized mass term. We shall choose the sign of this mass term to be negative so that H

develops a non-zero VEV on the brane. This is sufficient for forming the Z_{2N}-odd but $SU(2)\times U(1)$-invariant brane charge out of the two doublets.

To achieve this we shall require that the $SU(2)\times U(1)$-invariant product of the two doublets transforms under the brane "charge conjugation" Z_{2N} symmetry

$$(H^*\phi) \,\rightarrow\, e^{i\frac{\pi}{N}}(H^*\phi) \tag{63}$$

The new H-dependent terms in the action are

$$S \,=\, \int_{3+1} -\frac{1}{6}\,C_{\alpha\beta\gamma}J^{\alpha\beta\gamma}_{(T)} \,+\, M_{br}(x)^2|H|^2 \,+\, |D_\mu H|^2 \,-\, M_H^2\,|H|^2 \,-\, \frac{1}{4}|H|^4 \,+ ... \tag{64}$$

The current $J_{(T)}$ is given by (23) with

$$q_{eff} \,=\, \frac{\mu}{6}\,Re\left(\frac{H^*\phi}{M_P^2}\right)^N \tag{65}$$

The second term is the brane-localized mass term

$$M_{br}^2(x) \,=\, \int d\xi^3\sqrt{-g}\,M_{BH}\delta^4(x-Y), \tag{66}$$

where $M_{BH} \,>\, 0$. In addition the total action will contain all possible $SU(2)\times U(1)\times Z_{2N}$-invariant couplings among H, ϕ and F. These are unessential and are not shown for simplicity. The only requirement on such cross-interaction terms of the form $|H|^2|\phi|^2$, $|H|^2F^2$ is that they do not make the effective bulk mass^2 of H negative. This is easy to arrange by choosing the signs of these interactions, and can be assumed to be the case without any loss of generality. Note that Z_{2N} symmetry forbids the appearance of the mixing term $H^*\phi$ in the action.

As long as

$$M_{BH}^2 \,>\, 4M_H^2, \tag{67}$$

H develops a non-zero VEV on the brane. This is because for (67) the linearized equation for H in the brane background (located at $z=0$)

$$(\partial^2 \,+\, M_H^2 \,-\, M_{BH}\delta(z))H \,=\, 0 \tag{68}$$

has a localized exponentially-growing tachyonic mode

$$H \,=\, \mathrm{e}^{\frac{1}{2}t\sqrt{M_{BH}^2-4M_H^2}}\mathrm{e}^{-\frac{1}{2}M_{BH}|z|}. \tag{69}$$

Because H is non-zero on the brane, the effective brane charge (65) vanishes only for $\phi\rightarrow 0$, and vacuum $\phi=0$ is an attractor.

7 The doublet-triplet splitting

In Grand Unified Theories (GUTs) the low energy supersymmetry alone cannot guarantee the smallness of the Higgs mass, due to the problem of Doublet-Triplet splitting. The problem can be illustrated on an example of a simplest $SU(5)$-extensions of the standard model. Because of $SU(5)$ symmetry the Higgs doublet ϕ acquires a color-triplet partner (T), and the two together form the five-dimensional representation of the $SU(5)$ group, $5_{Higgs} = (\phi, T)$. Thus, because of GUT symmetry both the weak doublet and the color-triplet are forced to couple to quarks and leptons, that transform as $10 + \bar{5}$ dimensional representations per generation

$$5_{Higgs}\, 10\, 10 + \bar{5}^{*}_{Higgs} 10 \bar{5}. \tag{70}$$

$SU(3) \times SU(2) \times U(1)$-reduction of the above coupling shows that the three-level exchange of the color-triplet Higgs violates the baryon number and would mediate an unacceptably fast proton decay, unless T acquires a very large mass due to GUT symmetry breaking. In the same time ϕ should stay light, and this requires an additional fine tuning not provided by supersymmetry alone. Let Σ_i^j be an $SU(5)$-adjoint Higgs that breaks the GUT symmetry ($i, j = 1, 2, ...5$ are $SU(5)$-indexes). The doublet-triplet mass splitting is accomplished through the coupling of 5_{Higgs} with Σ

$$5^{*}_{Higgs}(a\Sigma^2 + b\Sigma)5_{Higgs} + (c\mathrm{Tr}\Sigma^2 + m^2)5^{*}_{Higgs}5_{Higgs} \tag{71}$$

where a, b, c, are some constants. After Σ develops the VEV $\Sigma = \mathrm{diag}(2, 2, 2, -3, -3)\sigma$ the masses of ϕ and T become split as

$$m_\phi^2 = (9a + 30c)\sigma^2 - 3b\sigma + m^2 \tag{72}$$

and

$$m_T^2 = (4a + 30c)\sigma^2 + 2b\sigma + m^2 \tag{73}$$

respectively. The additional fine tuning amounts to setting (72) to $\sim (100\mathrm{GeV})^2$.

We wish to point out now that the attractor solution of the Hierarchy Problem, automatically solves the problem of doublet-triplet mass splitting in Grand Unified Theories. Applying the attractor idea to GUTs simply amounts to promoting m^2 in (72) and (73) into the function of the four-form field-strength F. Then, just as in case of Standard Model, m_ϕ^2 is attracted to a small value, and in the same time m_T^2 is attracted to a large one. Note that depending of the parameters, because of $SU(5)$ symmetry of the brane charge, there may be other attractor points, e.g., at $m_T^2 = 0$.

8 Discussions and Outlook

The attractor solution is probably as close as the axion-type dynamical relaxation mechanism could come to the solution of the hierarchy problem. Indeed, the first

step of the attractor solution is exactly to bring the hierarchy problem on the same footing as the Strong CP Problem[13] in QCD. The latter, as we know, is not the problem of UV-sensitivity but rather the problem of the super-selection rule among the infinitely many θ-vacua. As we have shown in Section 3.1, this is precisely what happens to the Hierarchy Problem when we couple Higgs to the three-form field: From the problem of UV-sensitivity of the Higgs mass, it gets converted into a *super-selection problem* of the latter. The continuum of the vacua scanned by m_ϕ are all good, very much like QCD θ-vacua in QCD.

Having achieved this, we realize that there is a profound difference between the problem of UV-sensitivity and the problem of super-selection. The solution of UV-sensitivity problem always requires a new strongly-coupled physics at low energies, whereas the solution of the super-selection problem does not. In the former case the new strongly-coupled physics is required in order to regulate the quadratic divergency in the Higgs mass. An example for such a regulating physics is the law energy supersymmetry. Whereas, the new physics which solves the super-selection problem, can be *arbitrarily weakly* coupled. The good example of such new physics in case of the Strong CP problem, is the axion, which can have an arbitrarily high scale and be practically invisible. Like-wise, in our case the new physics that selected the attractor vacuum is arbitrarily decoupled, and can be practically unobservable at low energies. Nevertheless, as we have seen already the minimal realistic models can be predictive and potentially testable. Attractor are the mechanism through which the multi-vacua fundamental theory could make sharp low energy prediction.

If the attractor solution of the Hierarchy Problem can be fully implemented in String Theory, it will most likely have "softer" properties. For instance, the infinite number density of vacua probably will be regulated, as already suggested by the distribution of vacua in[4, 5]. This should not be an obstacle for solving the hierarchy problem, provided the number of vacua is sharply peaked around the small Higgs mass. Moreover, as we so in section 5.2, the softened attractors can appear already in effective field theory treatment.

In theories with attractor vacua there are number of open important questions, such as, whether some version of the attractor mechanism could select the small cosmological constant. Some of these questions will be addressed in [22]

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9 Appendix

9.1 Appendix A: Gauge Invariance from Goldstones

In this appendix we shall discuss how the coupling (22) can be obtained by integration of Goldstone-type degrees of freedom in a local, gauge-invariant theory. Let us first illustrate the idea on an example of electrodynamics. Imagine, that we wish to couple a photon A_μ to a non-conserved current J_μ ($\partial_\mu J^\mu \neq 0$) in a gauge invariant way. For example, such can be a $1+1$-dimensional version of the current (12)

$$J^\mu(x) \, = \, \int d\xi \delta^2(x \, - \, Y(\xi)) \, q(Y) \left(\frac{\partial Y^\mu}{\partial \xi} \right) \tag{74}$$

which in general is not conserved unless q is a constant. The conservation can be restored by introducing additional degrees of freedom, that will compensate the divergence of (74) for non-constant q.

This can be accomplished by introducing a compensating Stückelberg field θ. The coupling to the source then can be written in the following form

$$(A_\mu \, - \, \partial_\mu \theta) \, J^\mu \tag{75}$$

This coupling will be gauge invariant if we demand that under the gauge transformation

$$A_\mu \, \rightarrow \, A_\mu \, + \, \partial_\mu \omega \tag{76}$$

the compensating field shifts as

$$\theta \, \rightarrow \, \theta \, + \, \omega \tag{77}$$

Hence, θ is in fact a Goldstone field. Because of the gauge invariance, the Lagrangian can only depend on θ through the combination $(A_\mu \, - \, \partial_\mu \theta)$. Since current J_μ is not conserved, to maintain the gauge invariance we have to impose an additional constraint.

$$\partial^2 \theta \, = \, \partial_\mu A^\mu \tag{78}$$

Then, integrating out θ we can write down an effective Lagrangian for A_μ

$$L \, = \, -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \, + \, J^\mu \Pi_{\mu\nu} A^\nu \tag{79}$$

We see that after integrating out of the Stückelberg field, photon only couples to the transverse part of J_μ, as required by gauge invariance.

The generalization of the above construction to the case of the three-form field $C_{\alpha\beta\gamma}$ is straightforward. Again in order to achieve a gauge-invariant coupling of the three-form field to a non-conserved current $J_{\alpha\beta\gamma}$, we introduce a compensating two-form $B_{\alpha\beta}$, which under the gauge transformation (2) shifts in the following way

$$B_{\alpha\beta} \, \rightarrow \, B_{\alpha\beta} \, + \, \Omega_{\alpha\beta}, \tag{80}$$

The gauge invariant coupling to an arbitrary non-conserved source $J_{\alpha\beta\gamma}$ is

$$(C_{\alpha\beta\gamma} \,-\, F^B_{\alpha\beta\gamma})\, J^{\alpha\beta\gamma}, \tag{81}$$

where F^B is the field strength of B

$$F^B_{\alpha\beta\gamma} \,=\, d_{[\alpha} B_{\beta\gamma]} \tag{82}$$

As in the photon example, we have to impose the following constraint on B

$$\partial^\mu F^B_{\mu\alpha\beta} \,=\, \partial^\nu C_{\nu\alpha\beta} \tag{83}$$

This constraint can be enforced by introducing an additional auxiliary two-form field $X^{\beta\gamma}$ with the following coupling in the Lagrangian

$$X^{\beta\gamma}\partial^\alpha(C_{\alpha\beta\gamma} \,-\, F^B_{\alpha\beta\gamma}). \tag{84}$$

The equation of motion of $X^{\alpha\beta}$ then imposes the constraint (83). Now integrating out the $B_{\alpha\beta}$-field we write down the effective Lagrangian for C

$$L \,=\, -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \,+\, \frac{1}{6}J^{\mu\alpha\beta}\Pi^\nu_{[\mu}C_{\nu\alpha\beta]} \tag{85}$$

The above effective coupling coincides with (22) up to a total derivative.

9.2 Appendix B: The Charge Screening

We shall now discuss a potential effect, which may lead to the screening of the 3-form charge by the brane loops. The effect is somewhat similar in spirit to the charge screening by fermion loops in the massless Schwinger model[23], except that this issue in our case is more subtle, as we shall now discuss. For simplicity let us consider the analogous question in electrodynamics first. Consider the action given in (79) where J_μ is some generic current. Since the photon couples only to its transverse part, the theory is automatically gauge invariant regardless whether the current J_μ is conserved or not. Consider now a correlator of the two currents

$$\langle J_\mu(x)J_\nu(x')\rangle \,=\, P\Pi_{\mu\nu} \,+\, R\,\Theta_{\mu\nu} \tag{86}$$

where P and R are some scalar functions of the cut-off and the momentum. If the transverse part of the correlator is nonzero ($P \neq 0$), it will generate the following effective operator in the photon action

$$A^\mu P\Pi_{\mu\nu}A^\nu \tag{87}$$

Depending on the structure of the operator P this can be interpreted as the correction either to the photon kinetic terms, to the mass term, or to both. In case, if the propagator $\frac{1}{\partial^2+P}$ has a physical pole, the (87) generates the mass for photon, and

charges will be screened. This is in particular the case for P =constant. The result depends on the underlying structure of the theory. For instance, for the conserved fermionic current J^μ, (79) is equivalent to usual massless electrodynamics, which gives different answers in different number of dimensions. In $3+1$ dimensions, for the conserved fermionic current $P \propto \partial^2$, and no photon mass is generated. On the other hand in $1+1$ Schwinger model the answer depends on the fermion mass[23]. In case of massless fermions, the photon mass is generated and the charges are screened. For massive fermions, however, the screening is only partial.

Let us now show that the analogous question can be posed in our case. Consider the coupling (22). The correletor of the two currents J_μ can again be parameterized in the form (86). If $R \neq 0$ this will generate the following operator

$$C^{\mu\alpha\beta}\, R\, \Pi^\nu_{[\mu} C_{\nu\alpha\beta]} \tag{88}$$

which would signal the generation of mass, if $\frac{1}{(1+R)}$ had a physical pole. Because in general R is expected to be a function of ∂^2/M^2, the requirement of the absence of the physical poles below the cut-off scale reduces to the requirement of the absence of the constant part in R. In our case, R depends on the loops of the super-heavy branes, and will not be attempted to be calculated.

9.3 Appendix C: Exact Solutions

The effective equation eq(31) was derived in the approximation of constant ϕ per unit step. In reality, there will be a small back reaction on ϕ in each individual step, which will lead to the re-adjustment of the brane charge, and subsequent re-adjustment of the VEVs in the bulk. It is obvious that in the attractor neighborhood, this back reaction is negligible, but it is instructive to take it into account for the completeness.

So we shall now derive the effective bulk equation determining the VEV of ϕ without ignoring the variation of ϕ in an elementary step. For this, we shall solve the equation (26) and (27) without the simplifying approximations listed below them. Substituting the form (4), the eq(26) now becomes

$$\partial^\nu\left((1-|\phi|^2/M^2)F_0\right) \;=\; -\mu\Theta^{\nu z}\left[Re\left(\frac{\phi}{M_P}\right)^N \delta(z)\right] \tag{89}$$

or equivalently

$$\partial^2\left((1-|\phi|^2/M^2)F_0\right) \;=\; -\mu\partial_z\left[Re\left(\frac{\phi}{M_P}\right)^N \delta(z)\right] \tag{90}$$

The solution with the correct boundary conditions is

$$F_0(x_j,z) \;=\; \frac{\mu}{2M_P^N\left(1-\frac{|\phi(x_j,z)|^2}{M^2}\right)}\left[Re(\phi^N(z=0,f_i(x_j,z)))\theta(z) \;-\right. \tag{91}$$
$$\left. Re(\phi^N(z=0,f_i(x_j,-z)))\theta(-z) \;+\; f_0(M_P^N/\mu)\right].$$

where $\theta(z)$ is the step function, and $x_i,\ i = 0, 1, 2$ are the three remaining space-time coordinates parallel to the brane. For clarity, we have indicated the coordinate dependence, in order to stress how the values of the ϕ-field at different locations determine the bulk value of F_0. The functions $f_i(z)$ are such that $f_i(x_j, z)|_{z=0} \,=\, x_i$,

$$\partial^2\phi^N(z = 0, f_i(x_j, z))) \,=\, 0 \tag{92}$$

Note that the function $\phi^N(z = 0, x_i)$ is the value of ϕ^N at the brane location. f_0 is the integration constant. Thus, we see that for small ϕ the value of $F_0(z)$ in the bulk vacuum $z \neq 0$ is essentially determined by the value of the ϕ^N function *at the brane location* $z = 0$. For $z \neq 0$ the dependence of $F_0(z)$ on a local values of $\phi(z)$ is rather mild and has additional suppression factor ϕ^2/M_P^2.

For instance, for the background values that are funbctions of z and t only, the solution is

$$\begin{aligned} F_0(z,t) \,=\, & \frac{\mu}{2M_P^N\left(1-\frac{|\phi(z,t)|^2}{M^2}\right)}\,[(Re(\phi^N(0,(t-z))\,\theta(z)\,- \\ & Re(\phi^N(0,(t+z)))\theta(-z)\,+\,f_0(M_P^N/\mu)]. \end{aligned} \tag{93}$$

From here it is clear that the change of expectation value of ϕ on the brane triggers the corresponding change in ϕ, which propagates away from the brane at the speed of light. Existence of such waves indicates the presence of some "hidden" massless degrees of freedom. This is not surprising, since the degree in question is the Stückelberg field $B_{\mu\nu}$, which we have integrated out. In some sense, this degree of freedom in our case plays the role analogous to the one played by the invisible axion in the solution of the Strong CP problem in QCD.

Substituting the solution (93) into the equation (27), we get the following bulk equation for ϕ

$$\begin{aligned} \partial^2\phi \,-\, \Big(m^2 \,-\, & \frac{\mu^2}{8M^2M_P^{2N}\left(1-\frac{|\phi(x_j,z)|^2}{M^2}\right)^2}\,[(Re(\phi^N(0,(t-z))\,\theta(z)\,- \\ & Re(\phi^N(0,(t+z)))\theta(-z)\,+\,f_0(M_P^N/\mu)]^2\Big)\,\phi\,+\,\lambda\,|\phi|^2\phi \,=\, 0 \end{aligned} \tag{94}$$

Since in the vacua F_0 takes discrete values that between the two neighboring vacua are spaces by $\sim \phi^N$, for small ϕ the scanning becomes almost contineous. So the dynamics of the small-ϕ vacua can be studied by considering the appropriate values of the integration constant f_0. In small ϕ vacua this constant takes the values

$$f_0^2 \simeq 8m^2M^2 \tag{95}$$

Expanding (94) about such a vacuum, and noticing that the value of the ϕ^N determining F_0 is taken at the brane location, we see that left-interactions are weak in the attractor vacuum.

9.4 Appendix D: Resolving the Brane

We shall now resolve the structure of the brane, and show that it can arise in the effective low energy theory in form of an "axionic" domain wall[9] Such a possibility was suggested in [24, 1], but we shall review it here in more details for completeness. For this we shall introduce an "axion" field a, defined modulo 2π, with the decay constant f_a. First, let us show that in presence of the coupling between the axion and the three-form field $C_{\alpha\beta\gamma}$, the axionic domain walls become branes charged under C[10].

For demonstrating this, the possible couplings to the Higgs field ϕ plays no role and we shall ignore the latter for simplicity. Consider then the following Lagrangian

$$L = \frac{f_a^2}{2}(\partial_\mu a)^2 - V(a) - \frac{q}{12\pi}\partial_\alpha a C_{\beta\gamma\delta}\epsilon^{\alpha\beta\gamma\delta} - \frac{1}{48}F^2, \tag{96}$$

where $V(a)$ is the axion potential, which is required to be periodic under

$$a \to a + 2\pi n \tag{97}$$

The equations of motion are

$$\partial^\mu F_{\mu\nu\alpha\beta} = \frac{q}{2\pi}\partial^\mu a \epsilon_{\mu\nu\alpha\beta} \tag{98}$$

$$f_a^2\partial^2 a + V_a - \frac{q}{12\pi}\frac{F^{\mu\nu\alpha\beta}}{24}\epsilon_{\mu\nu\alpha\beta} = 0 \tag{99}$$

Integrating the first equation, we get

$$F_{\mu\nu\alpha\beta} = \frac{q}{2\pi}(a + 2\pi k)\epsilon_{\mu\nu\alpha\beta} \tag{100}$$

where k is an integer integration constant. Under shift symmetry (97) k changes as

$$k \to k + n \tag{101}$$

The relation (100) implies that any axionic domain wall, through which a changes by Δa effectively acquires a three-form charge

$$q_{eff} = \frac{q\Delta a}{2\pi} \tag{102}$$

The value of Δa through the elementary wall can be found by substituting the solution (100) in to eq(99). This gives the following effective equation for the axion

$$f_a^2\partial^2 a + V_a + \frac{q^2}{24\pi^2}(a + 2\pi k) = 0 \tag{103}$$

[9] The "axion" should not be confused with the one solving the Strong CP problem. The role of the former can be played by one of the axions appearing in string compactifications.

[10] At the level of the present discussion we shall treat both a and C as "elementary" objects, without specifying their possible origin from the fundamental underlying theory[11].

The value of the Δa through the elementary wall is determined by the distance between the neighboring minima of the effective potential:

$$V_{eff} \,=\, V(a) \,+\, \frac{q^2}{48\pi^2}(a \,+\, 2\pi k)^2 \tag{104}$$

For the small q and fixed k, the number of local minima is roughly $\sim V(a)_{max}/q^2$, where $V_{max}(a)$ is the maximal value of $V(a)$. The elementary step is $\Delta a \simeq 2\pi$. Thus, the branes acquire an effective three-form charge $q_{eff} \,=\, q$. The above counting of vacua is not surprising, since the model (96) can be viewed as the simplest four-dimensional generalization of the massive Schwinger model, in which there is an analogous counting of states[25].

Restoration of the ϕ-dependence of the brane charge, can now be done in a straightforward way, by promoting q into the function of ϕ given by (21). The gauge-invariant Lagrangian has the following form

$$\begin{aligned} L \,=\, \frac{f_a^2}{2}\,(\partial_\mu a)^2 \,-\, V(a) \,-\, \frac{q_{eff}(\phi)}{12\pi}\partial_\alpha a\,\Theta^\alpha_\mu C_{\beta\gamma\delta}\epsilon^{\mu\beta\gamma\delta} \\ -\,\frac{1}{48}F^2 \,+\, |\partial_\mu\phi|^2 \,+\, |\phi|^2\left(m^2 \,+\, \frac{F^2}{48M^2}\right) \,-\, \frac{\lambda}{2}\,|\phi|^4 \,+\, ... \end{aligned} \tag{105}$$

The equations of motions are

$$f_a^2\partial^2 a \,+\, V_a \,-\, \frac{q_{eff}(\phi)}{12\pi}\frac{F^{\mu\nu\alpha\beta}}{24}\epsilon_{\mu\nu\alpha\beta} \,+\, (q_{eff} - derivatives) \,=\, 0 \tag{106}$$

$$\partial^\mu\left((1+|\phi|^2/M^2)F_{\mu\nu\alpha\beta}\right) \,-\, \frac{q_{eff}(\phi)}{2\pi}\,\Theta^{\gamma\mu}\partial_\gamma a\,\epsilon_{\mu\nu\alpha\beta} \,+\, (q_{eff} - \text{derivatives}) \,=\, 0 \tag{107}$$

$$\partial^2\phi \,+\, \left(-\,m^2 \,-\, \frac{F^2}{48M^2}\right)\,\phi \,+\, \lambda\,|\phi|^2\phi \,+\, \frac{\mu N}{24\pi}\,\frac{\phi^{*N-1}}{M_P^N}\,\partial_\alpha a\,\Theta^\alpha_\mu C_{\beta\gamma\delta}\epsilon^{\mu\beta\gamma\delta} \,=\, 0 \tag{108}$$

and again ignoring the derivatives of q_{eff} and the terms of order ϕ^2/M^2 in the l.h.s. of eq(107), and integrating the equation for C, we get the following effective equation for ϕ -vacua

$$-\,(m^2 - (a \,-\, 2\pi k)^2\frac{\mu^2}{4\pi^2M^2}(Re(\phi^N/M_P^N)^2)\phi \,+\, \lambda\,\phi^3 \,=\, 0, \tag{109}$$

where a is defined from the equation

$$V_a \,-\, \frac{q_{eff}^2}{24\pi^2}(a \,+\, k) \,=\, 0 \tag{110}$$

as discussed above, for small q_{eff} the minima of the ϕ potential are at $a \,=\, 2\pi n$, which means that near the attractor point the ϕ-vacua are defined from the equation (31).

9.5 Appendix E: Axion-three-form couplings from massless scalars.

We shall now discuss how the axionic domain walls with Higgs-dependent three-form charges can be generated by integrating out some intermediate scalar fields, and show that this is only possible if the scalars in question are exactly massless. The idea is to start with the local coupling

$$q_{eff}\partial_\mu a J^\mu, \tag{111}$$

where J^μ is the gauge-invariant current, with the divergence

$$\partial_\mu J^\mu \,=\, c\,F \,+\, ... \tag{112}$$

where c is some constant. Then "integrating out" J^μ we write the efective coupling between a and C. In order to fulfill this program, we shall consider a toy $1+1$ dimensional example first. This is a $1+1$-dimensional electrodynamics coupled to two pseudoscalars, an axion a and an additional pseudoscalar χ. The Lagrangian is

$$\begin{aligned} L \,=\, \frac{1}{2}\,(\partial_\mu a)^2 \,+\, \frac{1}{2}\,(\partial_\chi a)^2 + \, q_{eff}\partial_\mu a \partial^\mu \chi \,-\, V(a) \,- \\ -\,V(\chi)\,-\,\tau\partial_\alpha\chi C_\beta \epsilon^{\alpha\beta}\,-\,\frac{1}{4}F^2 \end{aligned} \tag{113}$$

Here C_α is the electromagnetic vector potential, and τ is some constant. q_{eff}, which is understood to be the function of the Higgs VEV, will be treated as a constant for a moment. The reason for us to consider the above example, is that electric field in $1+1$ shares some properties with a three-form in $3+1$ dimensions. Just like the latter, the non-interacting $1+1$ electric fields has no propagating degrees of freedom, but its value can change in the presence of charges, which play the role analogous to 2-branes in four-dimensions.

Our aim is to start from the local Lagrangian (124), which for arbitrary q_{eff} is invariant under the axionic shift symmetry (97) as well as under the gauge symmetry

$$C_\alpha \,\rightarrow C_\alpha \,+\, \partial_\alpha\Omega, \tag{114}$$

which is an $1+1$-dimensional analog of (2). However, a and C do not couple directly, but through the "intermediate" field χ. In fact, $\partial_\mu a$ couples to the current $\partial_\mu\chi$ whose divergence is set by F. So we expect that after integrating out χ we arrive to ($1+1$-dimensional analog of) the coupling (96). This coupling should guarantee that axionic "walls", which in $1+1$-dimensions are just particles, acquire electric charges controlled by q_{eff}. We shall now check if this indeed is the case. The equations of motion are

$$\partial^2 a \,+\, q_{eff}\partial^2\chi \,+\, V_a \,=\, 0 \tag{115}$$

$$\partial^2\chi \,+\, q_{eff}\partial^2 a \,+\, V_\chi \,-\, \frac{\tau}{2}F_{\mu\nu}\epsilon^{\mu\nu} \,=\, 0 \tag{116}$$

$$\partial^\mu F_{\mu\nu} = \tau \partial^\alpha \epsilon_{\alpha\nu}\chi \tag{117}$$

The last equation is solved by

$$F_{\mu\nu} = \epsilon_{\mu\nu}\tau(\chi + \chi_0) \tag{118}$$

where χ_o is some constant. Substituting this result in eq(119), we get the following effective equation fo χ

$$\partial^2\chi + q_{eff}\partial^2 a + V_\chi + \tau^2\chi + \tau^2\chi_0 = 0 \tag{119}$$

For the special choice of

$$V_\chi = -\tau^2\chi - \tau^2\chi_0 \tag{120}$$

the system of equations is solved by

$$\chi = -q_{eff}a \tag{121}$$

where a satisfies the equation

$$\partial^2 a + \frac{1}{1-q_{eff}^2}V_a = 0 \tag{122}$$

Because of 2π periodicity of $V(a)$, the latter equation always has an axionic domain wall solution, through which a changes by 2π. According to (121) the corresponding change in χ is $\Delta\chi = -q_{eff}2\pi$ and according to (118) the change in F is $\Delta F = -q_{eff}\tau 2\pi$. Thus, axionic walls indeed acquire an electric charge. However, such a behavior is a peculiarity of the choice (120). In fact for no other choice of V_χ may axionic walls have a charge proportional to q_{eff}. Indeed, the fields at infinity on both sides of the wall must assume the constants values satisfying

$$V_a = 0 \tag{123}$$

and eq(120). Unless the latter is identically zero, the change of χ through the wall will be determined by the neighboring minima of (120), which contains no reference to q_{eff}. Hence, change of F will not be set by q_{eff} either.

The reason of why the choice (120) is the only possible one is easy to understand from the symmetry point of view. For such a choice the Lagrangian can be rewritten as

$$L = \frac{1}{2}(\partial_\mu a)^2 + \frac{1}{2}(\partial_\chi a)^2 + q_{eff}\partial_\mu a\partial^\mu\chi - V(a) - \tag{124}$$
$$-\frac{1}{4}(\tau\chi\,\epsilon_{\mu\nu} - F_{\mu\nu})^2$$

which has an exact shift symmetry

$$C_\mu \to C_\mu + b\epsilon_{\mu\nu}x^\nu, \quad \chi \to \chi - b/\tau \tag{125}$$

Due to this symmetry, after integrating out F, the action for χ cannot be anything other than the one of a scalar field with vanishing potential.

The fact why in the model (124) the existence of the branes with electric charge $\propto q_{eff}$ requires a very special choice of $V(\chi)$ can also be understood in the following way. In the limit $V(a) = V(\chi) = 0$ there are the two continuous shift symmetries.

$$a \rightarrow a + \text{constant} \tag{126}$$

and

$$\chi \rightarrow \chi + \text{constant} \tag{127}$$

The corresponding currents are

$$J_a^\mu = \partial^\mu a + q_{eff}\partial^\mu \chi \tag{128}$$

and

$$J_\chi^\mu = \partial^\mu \chi + q_{eff}\partial^\mu a \tag{129}$$

respectively. Only the second shift symmetry is "anomalous" and corresponding current is not conserved

$$\partial^\mu J_\chi^\mu = \frac{\tau}{2}F_{\mu\nu}\epsilon^{\mu\nu} \tag{130}$$

Hence, χ is the only speudo-Goldstone boson whose shift is tied directly to F. But shift of χ is determined by $V(\chi)$ which in general carries no information about q_{eff}. Only in the limit (120), when the "bare" mass of χ is exactly canceled by the "anomaly" contribution, shift symmetry (127) remains exact and shift in χ adjusts to the minimal step of a suppressed by the small charge q_{eff}.

It is usefull to view the above effect in fermionic language. Indeed, for $V(\chi) \propto \cos 2\sqrt{\pi}\chi$, the Lagrangian (124) is a bosonised version of the following fermionic Lagrangian

$$L = \frac{1}{2}(\partial_\mu a)^2 - V(a) + q_{eff}\partial^\mu a \epsilon_{\mu\nu}\bar{\psi}\gamma^\mu\psi + i\bar{\psi}\gamma_\mu D^\mu\psi - m\bar{\psi}\psi - \frac{1}{4}F^2 \tag{131}$$

In the limit $m \rightarrow 0$, $V(a) \rightarrow 0$ there are the two continuous axial symmetries (126) and

$$\psi \rightarrow e^{i\gamma_5\theta}\psi \tag{132}$$

But only the second is anomalous, and the corresponding speudo-Goldstone boson is the fermionic composite, and not a. The composite pseudo-Goldstone can be related to an elementary scalar χ via standard bosonisation[26]

$$\bar{\psi}\gamma_\mu\psi = \epsilon_{\mu\nu}\partial^\nu\frac{\chi}{\sqrt{\pi}} \text{ and } m\bar{\psi}\psi = m\kappa\cos 2\sqrt{\pi}\chi \tag{133}$$

where κ is a charge-related constant. The reason why the axial $U(1)$ symmetry cannot "see" a as its pseudo-Goldstone boson has to do with the peculiarities of

1 + 1 dimensions, and in particular the fact that the axial and vector currents are related through

$$\epsilon_{\mu\nu}\bar{\psi}\gamma^{\nu}\psi = \bar{\psi}\gamma_{\mu}\gamma^{5}\psi \tag{134}$$

Generalization of the model (124) to four-dimensions is straightforward and the results are essentially unchanged.

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